NOTATION

α, heat transfer coefficient, $W/(m^2 \cdot K)$; q, heat flux density, W/m^2; T, t, temperature of the medium, deg K; W, velocity of the medium, m/s; L, length of tube, m; d, diameter of tube, m; δ, thickness, m; R, radius, m. The dimensionless groups: $Re_f = \bar{g}l/r\mu$. Subscripts: v, vapor flow; f, film of liquid; 0, initial value of a parameter; int, internal; Δ, differences; s, saturation; d, drop.

LITERATURE CITED

1. V. P. Isachenko, Heat Transfer during Condensation [in Russian], Moscow (1977).
2. I. Tanasawa, and Y. Utaka, Trans. ASME, Ser. C, J. Heat Transfer, No. 3, 181-187 (1983).
3. V. P. Isachenko, S. A. Sotkov, A. P. Solodov, and T. O. Omarbekov, Teploenergetika, No. 10, 52-55 (1983).
4. J. M. Croix, B. Casset, and R. Camus, Elude de Condensation En Gouttes Par. Promoteurs Organiques. Department de Transferk Copuersion D'energie Service Des Transferts Thermiques SETRE, 08.03.1978.
5. V. G. Rifert, A. I. Sardak, and A. N. Tobilevich, Izv. Akad. Nauk SSSR, Energ. Trans. No. 4, 101-109 (1985).

SOLUTION OF THE INVERSE PROBLEM ON DETERMINING THREE FIBER COMPOSITE CHARACTERISTICS

I. V. Goncharov and V. L. Mikov UDC 536.24

A method is proposed to determine the characteristics of a bonded composite: the fiber and matrix heat conductivity coefficients and the heat transfer coefficient between them, from the solution of the inverse problem of heat conduction.

Extended utilization of bonded composite materials evokes the necessity to investigate heat propagation processes in such media. From the thermophysical viewpoint, these materials are quite definitely of heterogeneous configuration [1]. The matrix can be considered homogeneous and isotropic while the bonding fiber in a beam or rod in structure is highly anisotropic. Under nonstationary heat transfer conditions, different thermophysical characteristics (TPC) of the material components specify their distinct temperature, that appears especially strongly in the composite surface layer [2, 3].

A multitemperature theory of heat conduction [4] has been developed to model heat transport processes in heterogeneous media. Taking the average of the temperature field over the section of each component results in a system of interrelated heat conduction equations that is closed by using the Henry law that sets up a connection between the thermal flux density between the components q_{ij} and their mean temperatures

$$q_{ij} = \alpha (\hat{T}_i - \hat{T}_j). \tag{1}$$

The practical lack of data about the TPC of the components and α hinders extensive utilization of the multitemperature theory.

When producing methods and apparatus to determine fiber and matrix TPC the tendency to raise the informativity [5] that is achieved by the development of fast-response, highly productive methods of complex nature that give information about a set of properties from one experiment should be taken into account. The possibility is examined in this paper, of determining the fiber and matrix heat conduction coefficients as well as the heat transfer coefficient between them from thermograms of a pulse experiment (the "laser burst" method).

The "burst" method was developed to determine the thermal diffusivity and specific heat coefficients of homogeneous materials [6] under the assumption that the TPC of the material are independent of the temperature. At this time it is utilized to determine the effective thermal diffusivity coefficients of definite classes of heterogeneous media [7]. Distinctive features of the method are the rapidity of executing an experiment and the accuracy that is associated with determining the relative and not the absolute quantities. The time behavior of the relative temperature

---

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 58, No. 3, pp. 493-499, March, 1990. Original article submitted October 4, 1988.

 0022-0841/90/5803-0364$12.50

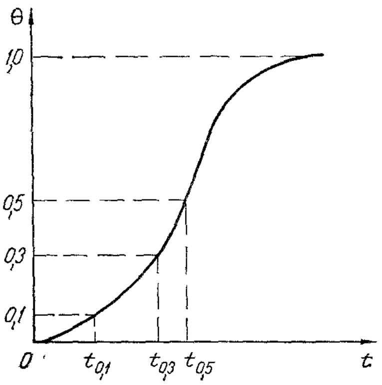

Fig. 1

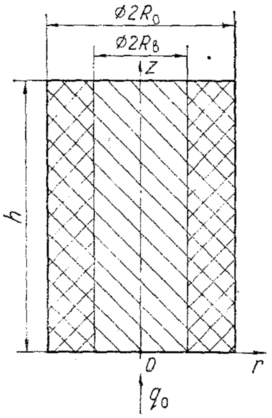

Fig. 2

Fig. 1. Characteristic thermogram of an impulse experiment.

Fig. 2. Representative volume of a unidirectional fiber composite.

of the specimen reverse surface after the action of a laser pulse on the forward surface can be obtained as a result of experiment (Fig. 1). The time dependence T(t) contains information about the TPC of the composite components, whose extraction demands the solution of the inverse problem of heat conduction.

Let us examine a unidirectional fiber composite and let us extract a representative volume containing one of the fibers and the matrix (Fig. 2). Under the assumption that the component TPC are independent of the temperature for the extracted representative volume, the following system of equations can be written ($i = 1, 2$)

$$c_i \frac{\partial T_i}{\partial t} = \lambda_{ri} \left(\frac{\partial^2 T_i}{\partial r^2} + \frac{1}{r} \frac{\partial T_i}{\partial r} \right) + \lambda_{zi} \frac{\partial^2 T_i}{\partial z^2}, \tag{2}$$

$$T_i(0,\ z,\ r) = 0, \tag{3}$$

$$T_1|_{R_в} = T_2|_{R_в}, \quad \lambda_{r1} \frac{\partial T_1}{\partial r}\bigg|_{R_в} = \lambda_{r2} \frac{\partial T_2}{\partial r}\bigg|_{R_в}, \tag{4}$$

$$\frac{\partial T_2}{\partial r}\bigg|_{R_0} = 0, \tag{5}$$

$$-\lambda_{zi} \frac{\partial T_i}{\partial z}\bigg|_{z=0} = \begin{cases} q_0, & 0 < t \leqslant t_u, \\ 0, & t > t_u \end{cases}, \quad \frac{\partial T_i}{\partial z}\bigg|_{z=h} = 0. \tag{6}$$

Let us average the temperature over the section for each component

$$\hat{T}_1(t,\ z) = \frac{2}{R_в^2} \int\limits_0^{R_в} T_1(t,\ z,\ r)\, r dr,$$

$$\hat{T}_2(t,\ z) = \frac{2}{R_0^2 - R_в^2} \int\limits_{R_в}^{R_0} T_2(t,\ z,\ r)\, r dr.$$

Then (2) takes the form

$$\begin{aligned} \lambda_{z1} \frac{\partial^2 \hat{T}_i}{\partial z^2} - c_1 \frac{\partial \hat{T}_1}{\partial t} &= \frac{2}{R_в} q^*, \\ \lambda_{z2} \frac{\partial^2 \hat{T}_2}{\partial z^2} - c_2 \frac{\partial \hat{T}_2}{\partial t} &= -\frac{2R_в}{R_0^2 - R_в^2} q^*, \end{aligned} \tag{7}$$

where

$$q^* = -\lambda_{r1} \left.\frac{\partial T_1}{\partial r}\right|_{r=R_B}.$$

The Henry law [4] can be utilized in a linear approximation, and by substituting (1) into (7) we arrive at a system of interrelated equations:

$$\lambda_{zi} \frac{\partial^2 \hat{T}_i}{\partial z^2} - c_i \frac{\partial \hat{T}_i}{\partial t} = (-1)^{i+1} \alpha_i (\hat{T}_1 - \hat{T}_2), \ i = 1, 2, \tag{8}$$

where $\alpha_1 = 2\alpha/R_B$; $\alpha_2 = 2\alpha R_B/(R_0^2 - R_B^2)$.

By using the Laplace transform in the time and the Fourier cosine transform in z the system of differential equations (8), (3), and (6) can be reduced to a system of algebraic equations for which, when solved, we have ($i = 1, 2$; $1 \neq j$)

$$\tilde{T}_i = q_0 [1 - \exp(-pt_u)] (2\alpha_0 + \lambda_{zj} \psi + c_j p)/(p\Gamma), \tag{9}$$

where

$$\Gamma = c_1 c_2 \{(p+d)^2 - f^2\}; \ d = B_0 + \psi a_0; \ B_0 = \frac{B_1 + B_2}{2};$$

$$a_i = \frac{\lambda_{zi}}{c_i}; \quad 2\alpha_0 = \alpha_1 + \alpha_2; \ a_0 = \frac{1}{2}(a_1 + a_2); \ B_i = \frac{\alpha_i}{c_i};$$

$$f^2 = (\psi_\Delta a + {}_\Delta B)^2 + B_1 B_2; \ 2_\Delta a = a_1 - a_2; \ 2_\Delta B = B_1 - B_2;$$

$$\tilde{T}_i = \int_0^\infty \exp(-pt) \left[\int_0^h \hat{T}_i(t, z) \cos\left(\frac{n\pi}{h} z\right) dz\right] dt; \quad \psi = \left(\frac{n\pi}{h}\right)^2.$$

Applying the inverse Laplace and Fourier transforms to (9) we obtain the averaged relative temperature on the specimen reverse surface

$$\Theta = 1 + \frac{1}{t_u}[\exp(-2B_0 t)(\exp(2B_0 t_u) - 1)]\,{}_\Delta C/(2B_0) + \sum_{n=1}^{\infty}\left\{\frac{(-1)^n}{f c_B t_u} \times \right.$$
$$\left. \times \left[\sum_{k=1}^{2} \exp(-v_k t)[\exp(v_k t_u) - 1] \frac{(-1)^{k+1}}{v_k}(2\alpha_0 + \hat{\lambda}\psi - v_k \hat{c})\right]\right\}, \tag{10}$$

where

$$\Theta = T_s/T_m; \ T_s = \hat{T}_1(t, h)\nu + \hat{T}_2(t, h)(1-\nu);$$

$$T_m = \frac{q_0 t_u}{h c_{ef}}; \ c_{ef} = [c_2(R_0^2 - R_B^2) + c_1 R_B^2]/R_0^2; \quad \nu = \left(\frac{R_B}{R_0}\right)^2;$$

$${}_\Delta c = c_{ef}[c_2 \nu + (1-\nu) c_1]/(c_1 c_2) - 1; \ c_B = c_1 c_2/c_{ef};$$

$$v_k = d + (-1)^k f, \ k = 1, 2; \ \hat{\lambda} = \lambda_1(1-\nu) + \lambda_2 \nu;$$

$$\hat{c} = c_1(1-\nu) + c_2 \nu.$$

As $t_u/t_{0.5} \to 0$ it is possible to simplify (10)

$$\Theta = 1 + \exp(-2B_0 t)\,{}_\Delta c + \sum_{n=1}^{\infty}(-1)^n (f c_B)^{-1}\left\{\sum_{k=1}^{2} \exp(-v_k t) \times \right.$$
$$\left. \times (2\alpha_0 + \hat{\lambda}\psi - v_k \hat{c})(-1)^{k+1}\right\}.$$

We consider the specific heat of the components c_1, c_2 and the specimen geometric characteristic to be known. To determine λ_{z1}, λ_{z2} and α we write the functional

$$F(\lambda_{z1}, \lambda_{z2}, \alpha) = \frac{1}{N}\sum_{n=1}^{N} F_n = \frac{1}{N}\sum_{n=1}^{N}(\Theta_e^n - \Theta_p^n)^2,$$

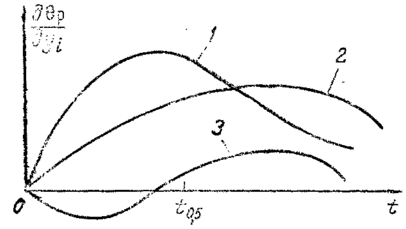

Fig. 3. Time dependence of the partial derivatives of the relative mean temperature of the specimen reverse surface with respect to the thermophysical characteristics y_i, $y = \{\lambda_{z1}, \lambda_{z2}, \alpha\}$: 1) $\partial\theta_p/\partial\lambda_{z1}$; 2) $\partial\theta_p/\partial\lambda_{z2}$; 3) $\partial\theta/\partial\alpha$.

where Θ_e^n is the experimental value of the relative mean temperature of the reverse surface of the specimen at the time t_n; Θ_p^n is the computed value of this quantity (according to (10)) for a certain set of desired coefficients $y = \{\lambda_{z1}, \lambda_{z2}, \alpha\}$. Determination of these coefficients reduces to a problem in minimization of the functional F [8].

Searching for the optimal vector $y_{op} = \{\lambda_{z1}^{op}, \lambda_{z2}^{op}, \alpha^{op}\}$ was realized by the Newton (NM) and Newton—Gauss (NGM) methods.

1. NEWTON METHOD

Three times t_1, t_2, t_3 are selected in which F_n is expanded in a Taylor series by limiting the latter to a linear approximation

$$F_n = 2(\Theta_p^n - \Theta_e^n)\left\{\sum_{j=1}^{3}\frac{\partial\Theta_p^n}{\partial y_j}\Delta y_j\right\},\ n = 1,\ 2,\ 3, \qquad (11)$$

or in matrix form $\left[\frac{\partial\Theta}{\partial y}\right]\Delta y = \Delta\Theta$, where

$$\left[\frac{\partial\Theta}{\partial y}\right] = \begin{Vmatrix} \frac{\partial\Theta_p^1}{\partial\lambda_{z1}} & \frac{\partial\Theta_p^1}{\partial\lambda_{z2}} & \frac{\partial\Theta_p^1}{\partial\alpha} \\ \frac{\partial\Theta_p^2}{\partial\lambda_{z1}} & \frac{\partial\Theta_p^2}{\partial\lambda_{z2}} & \frac{\partial\Theta_p^2}{\partial\alpha} \\ \frac{\partial\Theta_p^3}{\partial\lambda_{z1}} & \frac{\partial\Theta_p^3}{\partial\lambda_{z2}} & \frac{\partial\Theta_p^3}{\partial\alpha} \end{Vmatrix};$$

$$\Delta y = \begin{pmatrix} \Delta\lambda_{z1} \\ \Delta\lambda_{z2} \\ \Delta\alpha \end{pmatrix}; \quad \Delta\Theta = \frac{1}{2}\begin{pmatrix} (F_1)^{1/2} \\ (F_2)^{1/2} \\ (F_3)^{1/2} \end{pmatrix}.$$

Solving (11), we have $\Delta y_i = D_i/D$, where $D = \det\left[\frac{\partial\Theta}{\partial y}\right]$; and D_i is the determinant obtained from $\left[\frac{\partial\Theta}{\partial y}\right]$ by replacing elements of the i-th column by the column of free terms $\Delta\Theta$.

2. NEWTON-GAUSS METHOD (MODIFIED NEWTON METHOD)

When processing experimental data the NM can result in errors in the determination of the vector Δy. This is related to the fact that the dimensionality of y governs the quantity of experimental points by which the vector Δy is constructed. In our case the dimensionality of y is 3 and consequently three equations are used in the system (11). Evidently the more the experimental points that "operated" in the determination of Δy, the smaller will be the influence of experiment error on the convergence process. Let us expand the NM to a greater quantity of experimental points (N) as follows:

$$F_i = 2\,(\Theta_p^i - \Theta_e^i)\left\{\sum_{j=1}^{3} \frac{\partial \Theta_p^i}{\partial y_j}\,\Delta y_j\right\},\quad i = 1,\ 2,\ \ldots,\ N,$$

or in matrix form

$$\left[\frac{\partial \Theta}{\partial y}\right]_{(N\times 3)} \Delta \mathbf{y} = \Delta \Theta_{(N\times 1)}. \tag{12}$$

Multiplying (12) by the transposed matrix $\left[\frac{\partial \Theta}{\partial y}\right]^T_{(N\times 3)}$, we obtain

$$\left[\frac{\partial \Theta}{\partial y}\right]^T \left[\frac{\partial \Theta}{\partial y}\right] \Delta \mathbf{y} = \left[\frac{\partial \Theta}{\partial y}\right]^T \Delta \Theta,$$

and solving the system obtained for Δy, we represent the desired coefficients in the form $\Delta y_i = D_i^0/D^0$, where $D^0 = \det \left[\left[\frac{\partial \Theta}{\partial y}\right]^T \left[\frac{\partial \Theta}{\partial y}\right]\right]$, while D_i^0 is the determinant obtained from D^0 by replacing the i-th column by the column of free terms $\left[\left[\frac{\partial \Theta}{\partial y}\right]^T \Delta \Theta\right]$.

The vector $\Delta y = \{\Delta\lambda_{z1},\ \Delta\lambda_{z2},\ \Delta\alpha\}$ determines the direction of the descent. The search algorithm for the minimum of F has the following form:

1) initial values of y_0 are selected;

2) the Δy_k (k = 0, 1, 2, ...) is evaluated;

3) If k = 3n (n - 1, 2, ...), then we solve the optimization problem in the variable α for $\Delta\lambda_{z1} = \Delta\lambda_{z2} = 0$;

4) if k = 3n (n = 1, 2, ...), then we solve the minimization problem in β_k for the function $F(y_k + \beta_k \Delta y_k)$, whereupon we find the magnitude of the step β_k and the point $y_{k+1} = y_k + \beta_k \Delta y_k$;

5) we perform the comparison: if $F < \varepsilon$, then y_{k+1} is the solution of the problem, otherwise k = k + 1; we continue the search further, starting with No. 2.

Values computed by means of (10), i.e., from the solution of the direct problem, were used — to verify the inverse problem — in the capacity of the experimental data Θ_e^n.

Two model composites M1 and M2 were examined with the following TPC values; M1—λ_{z1} = 240 W/(m·K); λ_{z2} = 30 W/(m·K); $\alpha = 8.2\cdot 10^4$ W/(m²·K); M2 —λ_{z1} = 20 W/(m·K); λ_{z2} = 5 W/(m·K); $\alpha = 1.4\cdot 10^4$ W/(m²·K). The remaining TPC and the geometric parameters were taken identical: $c_1 = c_2 = 0.3\cdot 10^7$ J/(m³·K), h = 0.003 m, and $R_0 = 2R_B = 0.0012$ m.

Let us note that it is desirable to select the times $\{t_1, t_2, t_3\}$ in the NM in the segment $t_{0.1} = t_i = t_{0.3}$ (i = 1, 2, 3) (see Fig. 1). The time dependence of the derivatives with respect to the variables λ_{z1}, λ_{z2}, α for the function Θ is represented qualitatively in Fig. 3. All three derivatives $\partial\Theta/\partial\lambda_{zi}$, i = 1, 2; $\partial\Theta/\partial\alpha$ grow monotonically in the selected segment $[t_{0.1}, t_{0.3}]$, which favors convergence of the functional.

It is obtained for a broad class of initial data that when the NM is applied the convergence (to 1% accuracy for λ_{z1}, λ_{z2} and 2% accuracy for α) is assured for k = 4 (a computing time on the order of 1 min on an ES-1045 electronic computer). Analogous accuracy is achieved for the NGM for k = 8. In this case the computed values of Θ_e^i with rounding off after the fourth place were used. A more rapid convergence is achieved for initial data less than the desired, i.e., for "downward" motion.

1) $y_0 = (120;\ 42;\ 3.2\cdot 10^5) \xrightarrow{k=4} (241;\ 29.7;\ 8.35\cdot 10^4)$ (M1);

2) $y_0 = (150;\ 20;\ 3.5\cdot 10^5) \xrightarrow{k=4} (239.8;\ 30.05;\ 8.2\cdot 10^4)$ (M1);

3) $y_0 = (10;\ 2;\ 0.8\cdot 10^4) \xrightarrow{k=4} (20.3;\ 4.93;\ 1.43\cdot 10^4)$ (M2).

For "experimental" data Θ_e^i obtained with rounding off after the third place, the NGM has an advantage over the NM. For instance, for identical initial coefficients $y_0 = (31;\ 2,\ 0.3\cdot 10^4)$ the NM results in the point $y_4 = (19.8;\ 5.1;\ 1.2\cdot 10^4)$ (for the material M2); the error is 2% for values of the matrix heat conductivity and 11% for the heat elimination coefficient α. For the NGM the result is the point $y_6 = (19.8;\ 5.04;\ 1.33\cdot 10^4)$ and in this case the error is 1% for the matrix heat conductivity and 2% for α. Analogous results hold for the material M1 also. Therefore, when operating with experimental data the NGM will have advantages over the NM in connection with the utilization of greater information for the determination of the descent vector Δy_i.

This method can also be applied to laminar composites, for which the problem (2)-(6) should be solved in plane geometry. The form of α_i ($i = 1, 2$): $\alpha_i = \lambda_i/\delta_i$ changes here, where δ_i is the thickness of the i-th layer of the composite. For spatially bonded materials, partial homogenization of the medium should be performed to extract two components: the fibers whose axes are perpendicular to the specimen surface and the homogeneous medium consisting of the matrix and the remaining bond [2].

It is assumed in the mathematical model under consideration, as in the classical "burst" method, that the material TPC can depend sufficiently strongly on the temperature and replacement of the nonlinear nonstationary heat conduction model by a linear mathematical model results in errors in determining the desired characteristics. It is shown in [9] that the error originating in the determination of a by a classical method of "burst" can be 11.5%. Consequently, the proposed inverse problem can be applied for composites in a temperature band in which the component TPC depend weakly on the temperature. Unfortunately, quantitative criteria were not obtained for this temperature band.

Further development of the proposed method for the determination of TPC of composite components should be directed toward the development of a nonlinear nonstationary heat conduction model of the "burst" method with the temperature dependence of the component TPC taken into account.

NOTATION

z, r, space variables; t, time; $T_i(t, z, r)$, temperature of the i-th component; Θ, relative mean temperature of the reverse specimen surface; q_{ij}, thermal flux density from the i-th component to the j-th; α, coefficient of heat transfer between components; c_i, λ_{ri}, λ_{zi}, coefficients of volume specific heat, radial and axial heat conductivity; R_B, fiber radius; R_0, effective radius of matrix; h, specimen thickness; N, quantity of experimental points; q_0, heat flux density of the laser radiation; t_u, duration of the laser burst; p, n, Laplace and Fourier cosine transform parameters.

LITERATURE CITED

1. Yu. M. Tarnopol'skii, I. G. Zhigun, and V. A. Polyakov, Spatially-Bonded Composite Materials [in Russian], Moscow (1987).
2. I. V. Goncharov, Computation of the Temperature Field in Bonded Composite Materials [in Russian], Dep. No. 5389-B87 in VINITI July 29 (1987).
3. D. L. Balageas and A. M. Luc, Nonstationary Behavior of Directional Reinforced Composites. Limit of Application of Thermal Property Homogenization, AIAA Paper (ONERA) No. 1471 (1983).
4. L. P. Khoroshun and N. S. Soltanov, Thermoelasticity of Two-Component Mixtures [in Russian], Kiev (1984).
5. L. P. Filippov, Transport Phenomena [in Russian], Moscow (1986).
6. W. J. Parker, R. J. Jenkins, and C. P. Butler, J. Appl. Phys., 32, No. 9, 1679-1683 (1961).
7. R. Taylor, High. Temp.-High Press., 15, No. 2, 299-309 (1983).
8. O. M. Alifanov, Identification of Heat Transfer Processes of Flying Vehicles (Introduction to the Theory of Inverse Problems) [in Russian], Moscow (1979).
9. V. N. Kovtyukh, L. A. Kozdoba, and K. N. Lyubarskaya, Inzh.-Fiz. Zh., 46, No. 5, 769-773 (1984).